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ADMISSIBILITY OF TEST PROCEDURES BASED ON TWO PRELIMINARY TESTS FOR THE ANALYSIS OF GROUP OF EXPERIMENTS

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SUMMARY

A test procedure may have sufficiently large power and controlled size, but it can not be used unless it is admissible. In other words, admissibility of a test procedure is also an important and desired property. The admissibility of two test procedures, used in the analysis of group of experiments in mixed model, has been proved and the necessary and sufficient conditions for their admissibility have been derived.

Keywords : Mixed model; Central Chi-square; Orthogonal transformation.

Introduction

In testing of hypothesis, size and power are used as the main criterion for selecting an appropriate test procedure. In addition to these, it is also equally important to see whether the test procedure recommended for use is admissible or not. In case the test procedure is inadmissible, it becomes a compelling reason to discard it. Cohen [3] and Agarwal and Gupta [1] have derived necessary and sufficient conditions for admissibility of test procedures involving one PTS in random effects model and mixed effects model respectively. In the present paper, the necessary and sufficient condition for admissibility of two different test procedures involving two preliminary tests of significance (PTS) has been derived to analyse the data of group of experiments conducted at a number of places and for a number of years. The statistical model under study is a mixed model.

2. Description of the Problem

Consider a trial for t-treatments with r-randomized blocks, conducted at each of the p-places in y-years. Factor treatment is taken as fixed effect and place and years are taken as random effects. Therefore, the model under study is a mixed model. Let X_{ijkm} denotes the observation in mth block for kth treatment at jth place in ith year and it can be represented as follows :

$$X_{ijkm} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \delta_{ifm} + e_{ijkm} \quad (2.1)$$

where

$$i = 1, 2, \ldots, y$$
$$j = 1, 2, \ldots, p$$
$$k = 1, 2, \ldots, t$$
$$m = 1, 2, \ldots, r$$

Also $\sum_k \gamma_k = \sum_k (\alpha\gamma)_{ik} = \sum_k (\beta\gamma)_{jk} = \sum_k (\alpha\beta\gamma)_{ijk} = 0$, and e_{ijkm} are NID

$N(0, \sigma^2)$. It may be noted that

$\sum_i (\alpha\gamma)_{ik}$, $\sum_j (\beta\gamma)_{jk}$, $\sum_i (\alpha\beta\gamma)_{ijk}$ and $\sum (\alpha\beta\gamma)_{ijk}$

are not zero.

An abridged analysis of variance corresponding to model for testing a hypothesis about γ's is given in Table 1.

The interest is in testing the hypothesis $H_o : \sum_k (\gamma_k - \bar{\gamma})^2 = 0$ against $H_1 :$ $\sum_k (\gamma_k - \bar{\gamma})^2 > 0$. It is evident from analysis of variance Table 1 that no expected mean square can be used as error mean square for testing H_0 unless one of the two-factor interactions Place $\times$ Treatment and Year $\times$ Treatment is zero. Since any assumption about any of them to be zero will be arbitrary, it is in order to resort to the technique of PTS to ascertain their existence. An exact F-test for testing H_0 will be available as soon as one of these two interactions is zero. The existence of the interaction Year $\times$ Treatment is tested. Further, since the existence of second order interaction Year $\times$ Place $\times$ Treatment is also doubtful, it is first tested the preliminary hypothesis $H_{01} : \sigma^2_{ypt} = 0$ against $H_{11} : \sigma^2_{ypt} > 0$

TABLE 1—ABRIDGED ANALYSIS OF VARIANCE FOR GROUP OF EXPERIMENTS (MIXED MODEL)

Source of Variation	*d. f.*	*Mean Square*	*Expected Mean Square*
Treatment	$n_5 = t - 1$	V_5	$\sigma_5^2 = \sigma^2 + r\sigma_{ypt}^2 + ry\,\sigma_{pt}^2 + rp\,\sigma_{yt}^2 + rp\sigma_{ys}^2 + \frac{ryp}{t-1}\sum_k (\gamma_k - \bar{\gamma})^2$
Year × Treatment	$n_4 = (y-1)(t-1)$	V_4	$\sigma_4^2 = \sigma^2 + r\sigma_{ypt}^2 + rp\,\sigma_{yt}^2$
Year × Treatment	$n_3 = (p-1)(t-1)$	V_3	$\sigma_3^2 = \sigma^2 + r\sigma_{ypt}^2 + ry\sigma_{yt}^2$
Year × Place × Treatment	$n_2 = (y-1)(p-1)(t-1)$	V_2	$\sigma_2^2 = \sigma^2 + r\sigma_{ypt}^2$
Error	$n_1 = yp\,(t-1)(r-1)$	V_1	$\sigma_1^2 = \sigma^2$

by using the variance ratio V_2/V_1 and then the preliminary hypothesis $H_{02} : \sigma_{yt}^2 = 0$ against $H_{12} = \sigma_{yt}^2 > 0$ by using the ratio V_4/V_2 or V_4/V_{12} depending upon the outcome of H_{01}. A similar procedure would be obtained if $\sigma_{pt}^2 = 0$ is tested against $\sigma_{pt}^2 > 0$ after the testing of second order interaction and the only difference will be in the d.f. associated with the mean squares.

Keeping the above discussion in view, two test procedures, using Satterthwaite approximate F-statistics, have been formulated and studied for their admissibility. Each test procedure consists of four mutually exclusive situations under which main hypothesis H_0 is rejected.

TEST PROCEDURE I

Situation 1 :

$$\frac{V_2}{V_1} \geqslant F_1, \frac{V_4}{V_2} \geqslant F_2 \text{ and } \frac{V_5 + V_2}{V_4 + V_3} > F_3$$

Situation 2 :

$$\frac{V_2}{V_1} \geqslant F_1, \frac{V_4}{V_2} < F_2 \text{ and } \frac{V_5}{V_3} > F_4 \tag{2.2}$$

Situation 3 :

$$\frac{V_2}{V_1} < F_1, \frac{V_4}{V_{12}} \geqslant F_5 \text{ and } \frac{V_5 + V_{12}}{V_4 + V_3} \geqslant F_6$$

Situation 4 :

$$\frac{V_2}{V_1} < F_1, \frac{V_4}{V_{12}} < F_5 \text{ and } \frac{V_5}{V_3} \geqslant F_4$$

TEST PROCEDURE II

Situation 1 :

$$\frac{V_2}{V_1} \geqslant F_1, \frac{V_4}{V_2} \geqslant F_2 \text{ and } \frac{V_5}{V_4 + V_3 - V_2} \geqslant F_{32}$$

Situation 2 :

$$\frac{V_2}{V_1} \geqslant F_1, \frac{V_4}{V_2} < F_2 \text{ and } \frac{V_5}{V_3} \geqslant F_4 \qquad (2.3)$$

Situation 3 :

$$\frac{V_2}{V_1} < F_1, \frac{V_4}{V_{12}} \geqslant F_5 \text{ and } \frac{V_5}{V_4 + V_3 - V_{12}} \geqslant F_{62}$$

Situation 4 :

$$\frac{V_2}{V_1} < F_1, \frac{V_4}{V_{12}} < F_5 \text{ and } \frac{V_5}{V_3} \geqslant F_4$$

where

$$F_1 = F(n_2, n_1; \alpha_1),\ F_2 = F(n_4, n_2; \alpha_2)$$
$$F_3 = F(\nu_1, \nu_2; \alpha_3),\ F_4 = F(\nu_5, n_3; \alpha_4)$$
$$F_5 = F(n_4, n_{12}; \alpha_5),\ F_6 = F(\nu_3, \nu_2; \alpha_6)$$
$$F_{32} = F(\nu_5, \nu_4; \alpha_3),\ F_{62} = F(\nu_5, \nu_6; \alpha_6)$$
$$V_{12} = (n_1 V_1 + n_2 V_2)/n_{12},\ n_{12} = n_1 + n_2$$

It is known that the distribution of $n_i V_i/\sigma_i^2$ is central chi-square with d.f. n_i ($i = 1, 2, 3, 4$) and the distribution of $n_5 V_5/(C\ \sigma_1^2)$, by using Patnaik's [4] approximation, will be central chi-square with d.f. ν_5 where

$$\nu_5 = n_5 + 4\lambda^2/(n_5 + 4\ \lambda)$$

and λ is a non-centrality parameter given by :

$$\lambda = n_5\ (\theta_{15}^{-1} - 1)/2$$

and scale factor C is given by

$$C = 2 - \theta_{15}$$

The d.f. $\nu_i (i = 1, 2, 3, 4, 6)$ associated with $(V_5 + V_2)$, $V_4 + V_3)$, $(V_5 + V_{12})$, $(V_4 + V_3 - V_2)$ and $(V_4 + V_3 - V_{12})$ respectively are formulated by Satterthwaite's [5] approach and are given below :

$$\nu_1 = (C\nu_5\theta_{12}n_5^{-1} + 1)^2/(C\nu_5\theta_{12}^2 n_5^{-2} + n_2^{-1})$$

$$\nu_2 = (\theta_{14}^{-1} + \theta_{13}^{-1})^2/(n_4^{-1}\theta_{14}^{-2} + n_3^{-1}\theta_{13}^{-2})$$

$$\nu_3 = (C\nu_5 n_5^{-1} + 1)^2/(C^2\nu_5 n_5^{-2} + n_{12}^{-1})$$

$$\nu_4 = (\theta_{14}^{-1} + \theta_{13}^{-1} - \theta_{12}^{-1})^2/(n_4^{-1}\theta_{14}^{-2} + n_3^{-1}\theta_{13}^{-2} + n_2^{-1}\theta_{12}^{-2})$$

$$\nu_6 = (\theta_{14}^{-1} + \theta_{13}^{-1} - 1)^2/(n_4^{-1}\theta_{14}^{-2} + n_3^{-2}\theta_{13}^{-2} + n_{12}^{-1})$$

$$\theta_{ij} = \sigma_1^2/\sigma_j^2 (j = 2, 3, 4, 5)$$

In the above α_1, α_2, α_5 and α_3, α_4, α_6 are the levels of significance of PTS, final tests respectively.

3. Admissibility of Test Procedure I

The joint p.d.f. of V_i's $(i = 1, 2, \ldots, 5)$ belongs to a multivariate exponential family which is given as below :

$$K_0\left(\frac{n_1V_1}{\sigma_1^2}\right)^{a_1-1}\left(\frac{n_2V_2}{\sigma_2^2}\right)^{a_2-1}\left(\frac{n_3V_3}{\sigma_3^2}\right)^{a_3-1}\left(\frac{n_4V_4}{\sigma_4^2}\right)^{a_4-1}\left(\frac{n_5V_5}{C_5\sigma_1^2}\right)^{a_5-1}$$

$$\cdot \exp\left\{-\tfrac{1}{2}\left(\sum_{i=1}^{4}\frac{n_iV_i}{\sigma_i^2} + \frac{n_5V_5}{C_5\sigma_1^2}\right)\right\} \mathop{\pi}_{i=1}^{4} \frac{n_i}{\sigma_i^2} \; \frac{n_5}{C_5\sigma_1^2} \tag{3.1}$$

where K_0 is a constant.

Making the orthogonal transformation

$$W = TV \tag{3.2}$$

where

$$W' = (W_1, W_3, W_5, W_4, W_2)$$

$$V' = \left(\frac{n_1}{\sigma_1^4}V_1, \frac{n_2}{\sigma_2^4}V_2, \frac{n_3}{\sigma_3^4}V_3, \frac{n_4}{\sigma_4^4}V_4, \frac{n_5}{C_5\sigma_1^2\sigma_5^2}V_5\right)$$

and

$$T = \begin{bmatrix} \frac{4}{\sqrt{28}} & -\frac{3}{\sqrt{28}} & -\frac{1}{\sqrt{28}} & -\frac{1}{\sqrt{28}} & \frac{1}{\sqrt{28}} \\ \frac{3}{\sqrt{21}} & \frac{3}{\sqrt{21}} & \frac{1}{\sqrt{21}} & \frac{1}{\sqrt{21}} & -\frac{1}{\sqrt{21}} \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

in (3.1), we get the joint p.d.f. of w_i's of the exponential form

$$dP\,(W;\theta) = C(\theta)\, e^{-W\theta'}\, d\lambda(W)$$

where

$$C(\theta) = K_1\,(\sigma_1^2)^{a_1+2}\,(\sigma_2^2)^{a_2+1}\,(\sigma_3^2)^{a_3+1}\,(\sigma_4^2)^{a_4+1}\,(\sigma_5^2)^{a_5},$$

$$\theta = T\,(\sigma^2/2),$$

$$\sigma^{2\prime} = (\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2),$$

$d\lambda\,(W)$ is a function of W's and differential terms. The original main hypothesis H_0 reduces to the testing of $H_0 : \theta_3 = 0$ against $H_1 : \theta_3 > 0$.

It may be noted that the conditional distribution of W_5 given (W_1, W_2, W_3, W_4) belongs to one dimensional exponential family with parameter θ_3.

Suppose the test procedure I given by (2.2) is called $\phi\,(V)$ which we may write as $\phi\,(W)$ under transformation.

It will be admissible if and only if the acceptance region of $\phi\,(W)$ has convex section in W_5 for given (W_1, W_2, W_3, W_4), otherwise the sections of the critical region in W_5 for given (W_1, W_2, W_3, W_4) are half lines.

On applying the transformation (3.2), the various tests under test procedure I lead to the following inequalities :

$$\frac{V_2}{V_1} \geqslant F_1 \Rightarrow$$

$$W_5 \geqslant 2\left\{(4M_1 + 3)\frac{W_1}{\sqrt{28}} + 3\,(M_1 - 1)\frac{W_2}{\sqrt{21}}\right\} \qquad (3.3)$$

$$\frac{V_4}{V_2} \geqslant F_2 \Rightarrow$$

$$W_5 < 2\,(1 + M_2)^{-1}\left\{(2M_2 - 1)\frac{W_1}{\sqrt{28}} + (1 - 3M_2)\frac{W_2}{\sqrt{21}} - \frac{W_4}{\sqrt{6}} + \frac{W_3}{\sqrt{2}}\right\} \qquad (3.4)$$

$$\frac{V_5 + V_2}{V_4 + V_3} \geqslant F_3 \Rightarrow$$

$$W_5 \geqslant 2 \left\{ \frac{1}{n_2\theta_{12}^2} + \frac{C}{n_5\theta_{15}} + \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_3 \right\}^{-1}$$

$$\cdot \left[\left\{ \frac{3}{n_2\theta_{12}^2} - \frac{C}{n_5\theta_{15}} - \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_3 \right\} \frac{W_1}{\sqrt{28}} \right.$$

$$+ \left\{ - \frac{3}{n_2\theta_{12}^2} + \frac{C}{n_5\theta_{15}} + \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_3 \right\} \frac{W_3}{\sqrt{21}}$$

$$+ \left\{ - \frac{C}{n_5\theta_{15}} + \left(\frac{2}{n_3\theta_{13}^2} - \frac{1}{n_4\theta_{14}^2} \right) F_3 \right\} \frac{W_4}{\sqrt{6}}$$

$$\left. + \left\{ - \frac{C}{n_5\theta_{15}} + \frac{1}{n_4\theta_{14}^2} F_3 \right\} \frac{W_2}{\sqrt{2}} \right] \quad (3.5)$$

$$\frac{V_5}{V_3} \geqslant F_4 \Rightarrow$$

$$W_5 \geqslant - \frac{W_1}{\sqrt{7}} + \frac{2W_3}{\sqrt{21}} + \frac{2(2M_3 - 1)}{1 + M_3} \frac{W_4}{\sqrt{6}} - \frac{2W_2}{(1 + M_3)\sqrt{2}} \quad (3.6)$$

$$\frac{V_4}{V_{12}} \geqslant F_5 \Rightarrow$$

$$W_5 \leqslant 2\,(M_4 + \theta_{12}^2)^{-1} \left[\left\{ (3 - 4\theta_{12}^2)\, M_4 - \theta_{12}^2 \right\} \frac{W_1}{\sqrt{28}} \right.$$

$$\left. + \left\{ \theta_{12}^2 - 3\,(\theta_{12}^2 + 1)\, M_4 \right\} \frac{W_3}{\sqrt{21}} - \theta_{12}^2 \left(\frac{W_4}{\sqrt{6}} - \frac{W_2}{\sqrt{2}} \right) \right] \quad (3.7)$$

$$\frac{V_5 + V_{12}}{V_4 + V_3} \geqslant F_6 \Rightarrow$$

$$W_5 \geqslant 2 \left\{ \frac{1}{n_{12}\theta_{12}^2} + \frac{C}{n_5\theta_{15}} + \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_6 \right\}^{-1}$$

$$\cdot \left[- \left\{ \frac{4\theta_{12}^2 - 3}{n_{12}\theta_{12}^2} + \frac{C}{n_5\theta_{15}} + \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_6 \right\} \frac{W_1}{\sqrt{28}} \right.$$

$$+ \left\{ - \frac{3(\theta_{12}^2 + 1)}{n_{12}\theta_{12}^2} + \frac{C}{n_5\theta_{15}} + \left(\frac{1}{n_3\theta_{13}^2} + \frac{1}{n_4\theta_{14}^2} \right) F_6 \right\} \frac{W_3}{\sqrt{21}}$$

$$\left. + \left\{ - \frac{C}{n_5\theta_{15}} + \left(\frac{2}{n_3\theta_{13}^2} - \frac{1}{n_4\theta_{14}^2} \right) F_6 \right\} \frac{W_4}{\sqrt{6}} + \frac{F_6}{n_4\theta_1^{24}} \frac{W_2}{\sqrt{2}} \right]$$

(3.8)

where

$$M_1 = n_2\theta_{12}^2\, F_1/n_1$$

$$M_2 = n_4\theta_{12}^2\, F_2/(n_2\, \theta_{12}^2)$$

$$M_3 = n_5\theta_{15}\, F_4/(n_3\, C\, \theta_{13}^2)$$

$$M_4 = n_4\theta_{12}^2\, F_5/n_{12}$$

Denoting the right-hand side expressions of the above inequalities (3.3) to (3.8) by E_1, E_2, E_3, E_4, E_5 and E_6, respectively, rewrite them in the following form :

$$E_1 = k_1W_1 + k_2W_3 \tag{3.9}$$

$$E_2 = k_3W_1 + k_4W_3 - k_5W_4 + k_6W_2 \tag{3.10}$$

$$E_3 = k_7W_1 + k_8W_3 + k_9W_4 + k_{10}W_2 \tag{3.11}$$

$$E_4 = -k_{11}W_1 + k_{12}W_3 + k_{13}W_4 - k_{14}W_2 \tag{3.12}$$

$$E_5 = k_{15}W_1 + k_{16}W_3 - k_{17}W_4 + k_{18}W_2 \tag{3.13}$$

$$E_6 = -k_{19}W_1 + k_{20}W_3 + k_{21}W_4 + k_{22}W_2 \tag{3.14}$$

where k_1, k_2 are the coefficients of W_1, W_3 respectively in the inequality (3.3) and the respective coefficients of W_1, W_3, W_4, W_2 are k_3, k_4, k_5, k_6 in the inequality (3.4) and so on.

The acceptance region of $\phi(W)$ will be the union of the following four sets :

$$\begin{aligned} &W_5 : W_5 < \text{min.}\ (E_2, E_3) \cap W_5 > E_1 \\ &W_5 : W_5 < E_4 \cap W_5 > \text{max.}\ (E_1, E_2) \\ &W_5 : W_5 < \text{min.}\ (E_1, E_5, E_6) \\ &W_5 : W_5 < \text{min.}\ (E_1, E_4) \cap W_5 > E_5 \end{aligned} \tag{3.15}$$

The E_i's ($i = 1, 2, \ldots, 6$) can be represented by spheres with centres at the origin. The union of the four sets given by (3.15) under the following condition :

$$E_2 < E_5 < E_4 < E_1 < E_6 < E_3 \tag{3.16}$$

is a convex set. Hence the test procedure I is admissible.

4. Necessary and Sufficient Condition for Admissibility of Test Procedure I

Condition (3.16) may lead to the following inequalities after using the expressions for E's given by (3.9) to (3.14) :

$$E_2 < E_5 \Rightarrow W_1 < \frac{(k_{16} - k_4)\, W_3 + (k_5 - k_{17})\, W_4 + (k_{18} - k_6)\, W_2}{k_3 - k_{15}} \tag{4.1}$$

$$E_5 < E_4 \Rightarrow W_1 < \frac{(k_{12} - k_{16})\, W_3 + (k_{13} + k_{17})\, W_4 - (k_{14} + k_{18})\, W_1}{k_{11} + k_{15}} \tag{4.2}$$

$$E_4 < E_1 \Rightarrow W_1 > \frac{(k_{12} - k_2)\, W_3 + k_{13}\, W_4 - k_{14}\, W_2}{k_1 + k_{11}} \tag{4.3}$$

$$E_1 < E_6 \Rightarrow W_1 < \frac{(k_{20} - k_2)\, W_3 + k_{21}\, W_4 + k_{22}\, W_2}{k_1 + k_{19}} \tag{4.4}$$

$$E_6 < E_3 \Rightarrow W_1 > \frac{(k_{20} - k_8)\, W_3 + (k_{21} - k_9)\, W_4 + (k_{22} - k_{10})\, W_2}{k_7 + k_{19}} \tag{4.5}$$

$$E_2 < E_4 \Rightarrow W_1 < \frac{(k_{12} - k_4)\, W_3 + (k_5 + k_{13})\, W_4 - (k_6 + k_{14})\, W_2}{k_3 + k_{11}} \tag{4.6}$$

$$E_5 < E_1 \Rightarrow W_1 > \frac{(k_{16} - k_2)\, W_3 + k_{17}\, W_4 + k_{18}\, W_2}{k_1 - k_{15}} \tag{4.7}$$

$$E_4 < E_6 \Rightarrow W_1 > \frac{(k_{12} - k_8)\, W_3 + (k_{13} - k_9)\, W_4 - (k_{10} + k_{14})\, W_2}{k_7 + k_{11}} \tag{4.8}$$

Eliminating W_1, W_3, W_2 and W_4 in sequence from the inequalities (4.1) to (4.8) we get the necessary and sufficient condition for admissibility of test procedure I as follows :

$$(A_2 - A_1)(B_3 + B_4) > (A_4 - A_3)(B_1 + B_2) \tag{4.9}$$

where

$$A_1 = D_1^{-1} \{k_{13}(k_3 - k_{15}) - (k_5 - k_{17})(k_1 + k_{11})\}$$

$$A_2 = D_2^{-1} \{(k_{13} + k_{17})(k_7 + k_{19}) - (k_{21} - k_9)(k_{11} + k_{15})\}$$

$$A_3 = D_3^{-1} \{k_{21}(k_1 - k_{15}) + k_{17}(k_1 + k_{19})\}$$

$$A_4 = D_4^{-1} \{(k_{13} - k_9)(k_3 + k_{11}) - (k_5 + k_{13})(k_7 + k_{11})\}$$

$$B_1 = D_1^{-1} \{k_{14}(k_{15} - k_3) - (k_{18} - k_6)(k_1 + k_{11})\}$$

$$B_2 = D_2^{-1} \{(k_{14} + k_{18})(k_7 + k_{19}) + (k_{22} - k_{10})(k_{11} + k_{15})\}$$

$$B_3 = D_3^{-1} \{k_{22}(k_1 - k_{15}) - k_{18}(k_1 + k_{19})\}$$

$$B_4 = D_4^{-1} \{(k_{10} + k_{14})(k_8 + k_{11}) - (k_6 + k_{14})(k_7 + k_{11})\}$$

$$D_1 = (k_{16} - k_4)(k_1 + k_{11}) + (k_2 - k_{12})(k_3 - k_{15})$$

$$D_2 = (k_{20} - k_8)(k_{11} + k_{15}) + (k_{16} - k_{12})(k_7 + k_{19})$$

$$D_3 = (k_{16} - k_2)(k_1 + k_{19}) + (k_2 - k_{20})(k_1 - k_{15})$$

$$D_4 = (k_{12} - k_4)(k_7 + k_{11}) + (k_8 - k_{12})(k_3 + k_{11})$$

Remarks :

(1) Proceeding in the same way as in Sections 3 and 4 for test procedure I, it can be easily proved that test procedure II given by (2.3) is also admissible. Necessary and sufficient condition may be derived in a similar manner.

(2) There can be 6! inequality relations of the type (3.16) amongst E's and for a large number of relations the test procedures I and II are admissible.

(3) A's, B's are based on six F-values out of which only F_1, F_2 and F_5 may be chosen arbitrarily by the statistician so that the condition (4.9) holds good. Out of these admissible test procedures, the one which has largest power for the given size is selected.

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